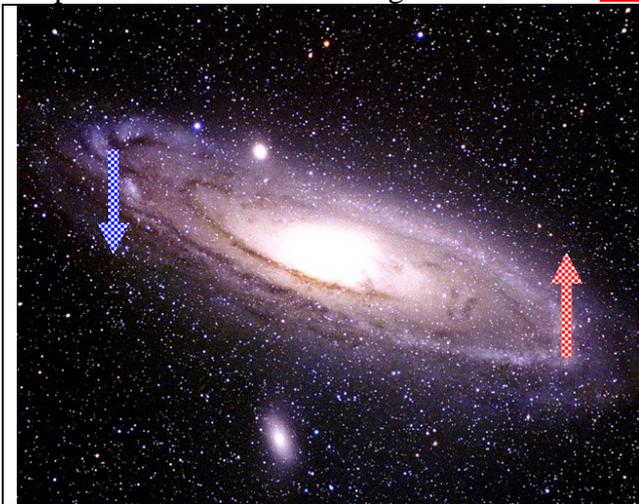


How to weigh a galaxy

The teaching activities consist in measuring the mass of a spiral galaxy, viewed edge-on, using the same procedure employed by astronomers. It is surprising how just a few measurements and the knowledge of a few fundamental laws of physics make it possible to weigh the largest and most distant objects in the Universe, despite the impossibility of carrying out direct measurements and the fact that the only information available to us derive from a few photons which have travelled for tens and tens of millions of years. What is even more surprising is the fact that the simple measurement proposed here makes it possible to obtain experimental evidence for the well-known *dark matter*.

Spiral galaxies contain a large amount of gas; the gas emits a spectrum of lines; if the galaxy is viewed *edge-on* (and not directly facing the observer, not perpendicular to the line of sight) and given that it rotates on its own axis, on one side the gas goes away from us and on the other, with respect to the centre, it draws closer to us (cf. figure). The lines emitted by a gas which moves with respect to the observer undergo the so-called Doppler effect, which shifts the observed frequency.



The measurement we shall make exploits the fact that the Doppler Shift is proportional to the velocity at which the gas moves away or draws nearer to us. We therefore obtain the velocity v of the gas as a function of its distance r from the centre of the galaxy. The plot of the velocity v of each point of the galaxy as a function of the centre r is known as the *rotation curve*. The velocity (assuming that the galaxy is in equilibrium and is subject to the Newtonian Law of universal gravity) is in turn proportional to the mass enclosed in the radius r .

This method is used by astronomers to calculate the mass of a large number of galaxies and is extremely important because it has made it possible to carry out systematic recording of an unexpected profile of the rotation curves, which is only explicable if it is assumed that the mass within the radius r far away from the galactic centre is much greater than the visible one, which is in other words the mass which emits radiation. This is better known as the famous problem of the *dark matter*, which today poses one of the most fascinating challenges to modern physics.

Propaedeutics

Continuous electromagnetic spectrum and spectrum of lines; Doppler¹ effect; classification and characteristics of the galaxies.²

¹ <http://lectureonline.cl.msu.edu/~mmp/applist/doppler/d.htm>

² Cf. ppt

Exercise

Phase 1 : Reduction of the Archive data

If you don't want to reduce the data and prefer to use data which has already been processed and is ready for use, click [here](#).

The new instruments available to astronomers, in particular CCD cameras, are becoming increasingly precise and efficient. However, it is still necessary to take certain precautions in order to take account of imperfections, systematic errors caused by instruments etc. In this section you can learn how to prepare the images so that they can be used for scientific measurements etc. Here are the steps you should follow:

- 1.1 Editing the images (*p. xx*);
- 1.2 Creation of the master frames (*p. xx*);
- 1.3 Removal of the bias (*p. xx*);

- 1.4 Removal of the dark frame (*p. xx*);
- 1.5 Division by flat field (*p. xx*);
- 1.6 Sky subtraction (*p. xx*);

Phase 2 : Calibration in wavelengths

The CCD cameras provide images that preserve the spatial information: however, it is necessary to calibrate the instrument in order to understand what difference in wavelengths corresponds to the distance in pixels between two lines.

- 2.1 Identification of the lines in the calibration lamp (*p. xx*);
- 2.2 Relationship between pixel and wavelength (*p. xx*);
- 2.3 Calculation of the shift $\Delta\lambda$ as a function of the distance from the centre of the galaxy (*p. xx*);

Phase 3 : Calculation of the rotation curve

The Doppler Shift of the lines in the gas spectrum at various distances r from the centre of the galaxy can be used to work out velocity at these distances

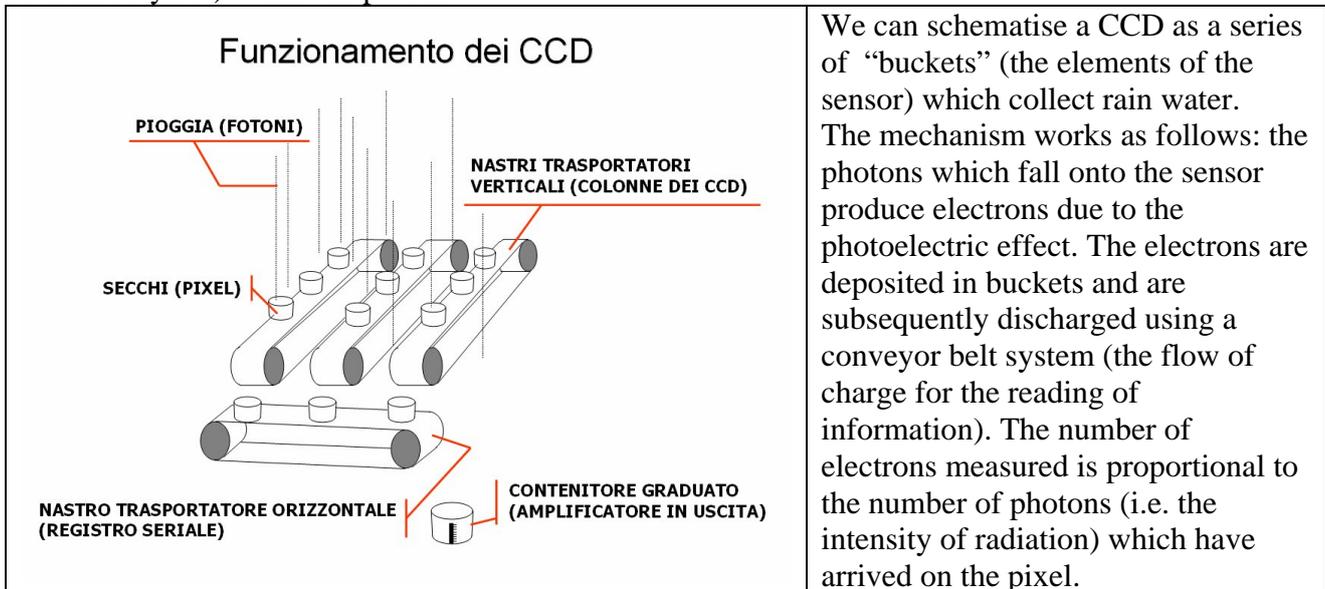
- 3.1 Calculation of the instrumental rotation curve (*p. xx*);
- 3.2 Application of the formula of the Doppler effect (*p. xx*);
- 3.3 Relation $v_{\text{oss}} = v_{\text{rot}} \sin(i)$; (*p. xx*)

Phase 4 : Calculation of the mass of the galaxy

Using the relation $\frac{GmM}{r^2} = \frac{mv^2}{r}$; (*p. xx*)

Phase 1 : Reduction of the archive data

A CCD detector consists of a matrix (x,y) of elements which are sensitive to light (though not exclusively so!) known as pixels.



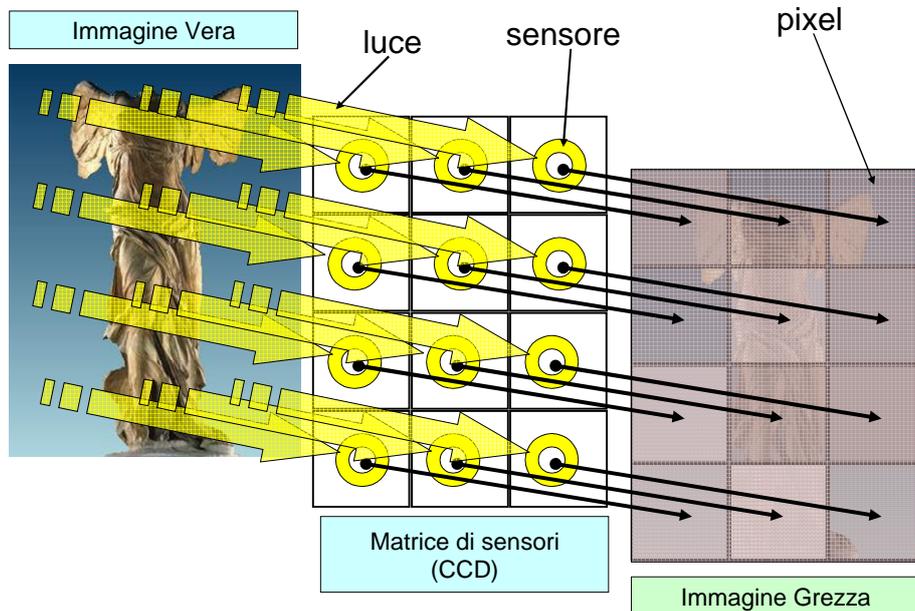
The signal recorded on exit from the CCD is a function $U(P)$, which depends on the point of the matrix (on the pixel $P(x,y)$); the function $U(P)$ is obviously linked to the entrance signal, though it is not exactly equal to it for two main reasons: first of all, because the instrument is capable of recording the entrance signal (which we shall refer to as $I(P)$) with a level of efficiency which is not the same for all pixels; the level of efficiency, which we shall term $A(P)$, is a multiplicative term which reduces the measured value; it depends on the CCD and the higher the quality of the CCD, the higher the value of the efficiency which approaches an ideal value, $A(P)= 1$.

Secondly, there are some factors which create *noise* $R(P)$; in other words, they create a signal which is added to the measurement. The first factor is the *bias* $B(P)$, a signal which is added prior to measurement in order to read weaker signals. The *bias* is estimated by taking a reading of the CCD with the shutter closed and a zero exposure time. Furthermore, if we make a measurement keeping the shutter closed (due to which the entrance signal $I(P)$ is definitely null), after a certain exposure time, we nevertheless record a different value from the *bias*, due to the fact that the sensor is relatively hot, causing its elements to emit electrons on account of thermic noise, which disturbs the reading. This is referred to as *dark current* $D(P)$. In order to reduce this effect, the CCD is generally cooled as much as possible: the CCD used for archive images is cooled with liquid nitrogen to a temperature of $-130\text{ }^{\circ}\text{C}$.

Another source of noise is introduced by the electronics of CCD (which are necessary to read the image from the CCD) $N(P)$. This term cannot be eliminated, even though it can be taken into account using appropriate filtering techniques.

By summarising these points, it can be said therefore that the exit signal can be calculated by the sum of the noise (bias plus dark current) and by the entrance signal multiplied by efficiency $A(P)$:

$$U(P) = A(P) \times I(P) + [B(P) + D(P)] + N(P)$$



In order to calculate the entrance signal, one must therefore process the astronomic image by subtracting the noise and dividing it by efficiency: this procedure is known as image *reduction*.
In other words one needs to make a series of measurements:

- **images of the celestial body (spectrum of the galaxy in this case)**

in our case, it is a spectrum; it is therefore the result of the decomposition of the light of the galaxy into its various components at different wavelengths, obtained due to the dispersing element incorporated in the telescope and recorded by the CCD.

Figure

- **bias images (term $B(P)$ in the above formula)**

The bias images are obtained by making the CCD work with the shutter of the telescope closed and the zero exposure time (in other words, the minimum exposure time possible for the instrument; it is like the minimum exposure time we can set for photo-cameras!)

- **dark current images (term $D(P)$ in the formula)**

Thermic noise is present in any sort of conductor; it is caused by thermic disturbance of the charge bearers and therefore behaves like a random current. This phenomenon also occurs inside the sensors of a CCD placed in total darkness, producing so-called **dark current**. Its effect is to reduce the minimum exposure time of the CCD: after a long time, the dark current would completely saturate the sensor, which could no longer accumulate electrons generated by the incident light. Therefore the lower the dark current, the longer are the exposure times that can be carried out with a given CCD. The dark images are obtained with the telescope shutter closed and an exposure time equal to that used to obtain the raw image.

It can be observed that the images thus obtained are nevertheless affected by read-out noise and by the Bias which therefore need to be subtracted.

- **flat field frames (term $A(P)$ of the formula)**

The CCD sensors are like snow flakes: no two are exactly alike. These differences mean that, even when all the pixels of the CCD matrix are illuminated, you can obtain an image with

lighter or darker pixels with respect to a mean value. Especially for longer exposures, the differences in the response (efficiency) between different sensors become an important factor. To make the whole image uniform we therefore divide it by the *flat field*, which is the image produced by projection onto a screen illuminated uniformly by a lamp. The flat field image must also be corrected for the *Bias*

N.B. the dark, bias and flat images are created using the same instruments (grating, slit, CCD etc) and under the same conditions (same night, same exposure times – if different from the minimum – etc) of the spectrums of the galaxy (or in general of the images which are the subject of astronomical observation)

Another image which is essential for our measurements is the so-called image of the calibration lamp, which needs to be used to convert the position of the pixels into wavelengths. This will be discussed below.

All the operations which need to be carried out on the images are described in detail in the following sections: it should be noted that the operations on the images are done using a special software which analyses the image pixel by pixel. For example, in the case of a subtraction between two images, the value recorded by the pixel in the position (x_1, y_1) of one image is subtracted from the value registered by the pixel in the same position (x_1, y_1) of the second image.

1. Cutting the images

[In-depth analysis (If you use the archive images, this operation is not necessary, because they all have the same dimensions)

Before being processed, the images must be “prepared” beforehand.

All the operations necessary for the reduction are carried out by superimposing and comparing the various images. However, in order to superimpose two images, they must be of exactly the same size.

Therefore the first step is to check the size of all the archive images and then cut them so that they are all equal.

In order to check the size of the available images, go to the *image* menu and click on the *show info* option, and then read off and note down the values of **NAXIS1** and **NAXIS2**, which correspond to the number of pixels, for x and y respectively, which make up the image.

If there are images which differ from one another in size, then it is necessary to adjust the size so that they correspond to size of the smallest image. To cut them, choose the rectangular section from the toolbar and select a rectangle of the desired size on the image. The size you have selected will appear on the left of your toolbar. Once you have obtained the right size, select *crop* from the *image* menu.]

1.b Creation of the master frames

The master frames are images obtained from the average of the images of the same type, taken in the same conditions. During the image acquisition phase, the observer may encounter phenomena over which he/she has no control, but which are unlikely to occur twice in succession (random errors); the average ensures that such random effects are eliminated.

To create the Master Bias, we need to take all the available Biases (N.B.: they must be the same size) and calculate the average. This same operation needs to be done to create a Master Dark image. The Master Flat image will already be provided in the archive data.

To create the *Master Frames* proceed in the following way:

1. open the images for which you want to calculate the average, choosing the *open* option from the *file* menu;
2. select the images to be averaged from those available on the list;
3. using the *operations* menu, select *operations on images*;
in the new window that will open, among the various proposed mathematical operations, choose the *averaging* operation;
4. the new image obtained must be saved by selecting, using the *file* menu, the *save as* option and then imposing the *FITS* format.

1.c Subtraction of the master bias

As mentioned previously, the Bias is a disturbance which must therefore be subtracted from ALL the other available images: Spectrum, Dark, Flat, Lamp.

To perform this subtraction :

1. open the image of the *Master Bias* created previously;
2. open the image from which one wants to subtract the *Master Bias* image;
3. choose the *operations on images* option from the *operations* menu;
4. select *subtract* from the various options proposed.

N.B. : the MBF must be subtracted from the other images and not vice versa !!!

1.d Subtraction of master dark

Open the Master Dark corrected image from which the Master Bias has been subtracted, and subtract it, as explained above, from the Spectrum, Flat and Lamp corrected images, from which the master Bias has been subtracted.

1.e Division by the master flat

Lastly, the spectrum and the lamp images, which have been corrected for master bias and master dark, must be divided by the master flat.

1. Open the spectrum and lamp image corrected in point 1.d
2. Open the master flat
3. Select the *operations among images* option from the *operations* menu, and select *divide*.