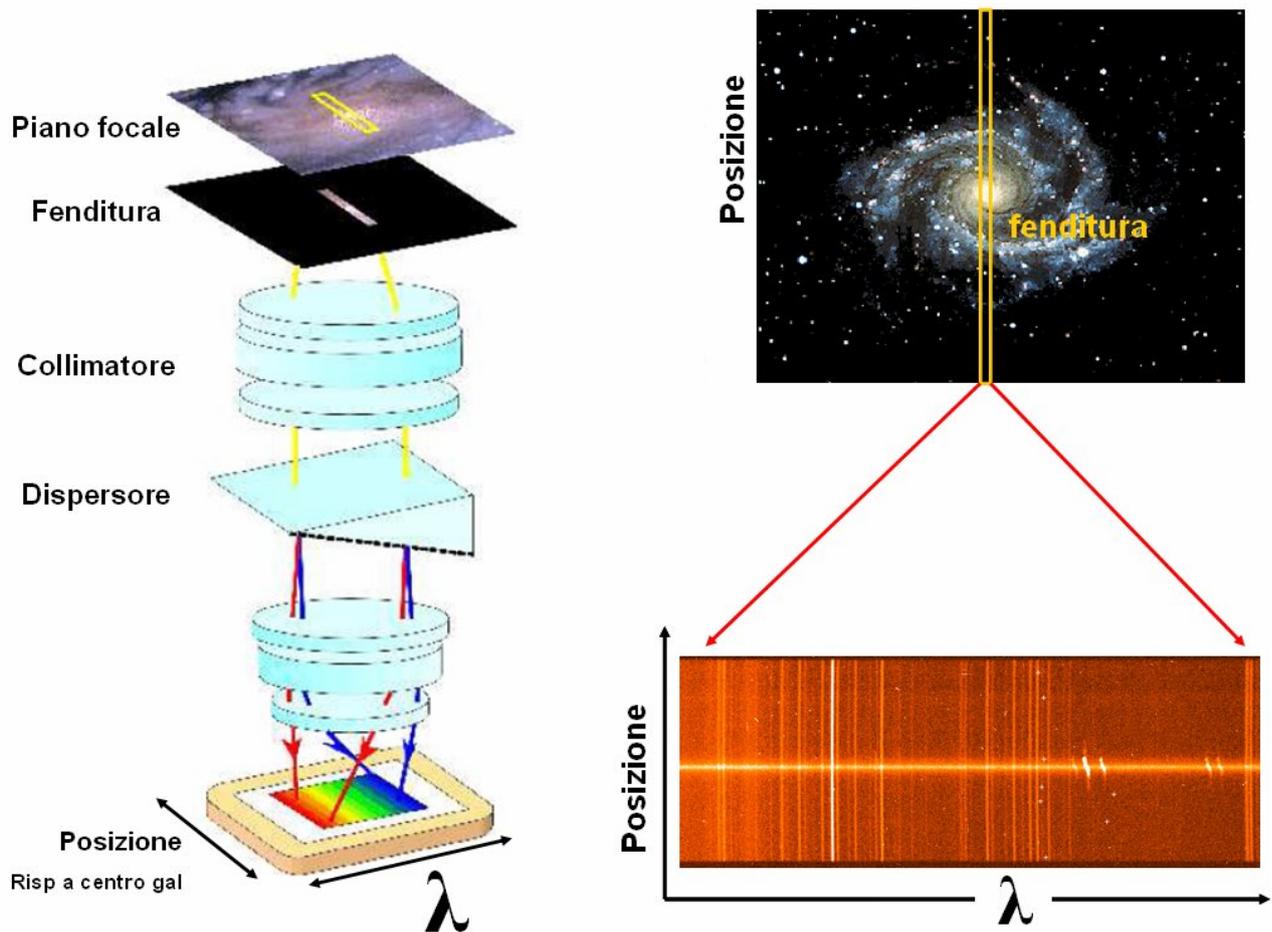# Phase 2 : Calibration in wavelengths

The focal point of the telescope is the point at which the image is formed. The eye or a CCD placed in the focal plane can reveal the image of the celestial body (e.g. the image of the galaxy in the top right corner). In order to obtain the spectrum of the observed object, you place a slit in the focal plane which selects a specific "slice"; the radiation from this slice is passed through a dispersant element (prism, grating etc.) which separates the wavelengths creating the spectrum of the galaxy recorded by the CCD camera. In this way, we obtain an image which, along the *x* axis, contains information about the contents in λ of the radiation from various points of the slice; for a specific wavelength, the contributions of points in different positions are shown in the *y* direction. In the image of the spectrum, the horizontal line at the centre represents the emission from the centre of the slice, i.e. from the nucleus of the galaxy, which clearly does not just emit lines at a specific λ, but a continuum at all the (recordable!) λs; the vertical lines which cross the entire spectrum are caused by the emission of the earth's atmosphere; this emission does not change with variations in the distance from the centre of the galaxy (along the *y* axis). If the slit also captured the light from a field star, we would see another line parallel to that of the nucleus (continuous for all the λ), situated for example beneath it if the emission derived from a region beneath the galaxy.

2.a Identification of the lines in the calibration lamp

Together with the spectrums and images for reduction (dark, bias, flat etc..), the archives also provide the spectrum of a "calibration lamp".
This spectrum is used to determine the relationship which links the wavelength and the position in pixels in images taken on our instrument.
Calibrating the spectrum means transforming the positions in pixels in the image into wavelengths. To do this, we use the spectrum of a lamp which emits lines of known wavelengths; these lines will fall in precise positions, and will enable us to establish the wavelength to which corresponds each pixel in our image because the lines of the lamp have known wavelengths!

1.   Open the image of the lamp, and select the key *select straight line* from the tool bar.
2.   Plot a horizontal line at the centre of the image of the lamp from left to right and choose the *profile* option from the *analyse* menu.
3.   By comparing this graph with the image of the lamp, you will notice that as you move the graph with the cursor, a yellow dot
 moves simultaneously on the image. Use this to identify two extremely dense lines in the lamp and determine which "peaks" these lines correspond to in the graph.
4.   Measure the distance in pixels, Δpixel, of the two chosen lines, plotting a *select straight line* between them and selecting the *measure* option from the *analyse* menu. Note down this measurement, because this is the distance in pixels between the lines, which will then be converted into wavelengths.
5.   Now compare your graph with the one provided by the archive, and note down the wavelengths corresponding to the lines that you chose previously: the difference between these will be your $\Delta\lambda$.

$$\Delta\text{pixel} \quad = \quad \dots\dots\dots\dots$$

$$\Delta\lambda \quad = \quad \dots\dots\dots\dots$$

2.b Relationship between pixels and wavelengths

Let's discover the distances in pixels and in λ between two lines of the lamp, and we will use these to calibrate the spectrum of the galaxy.

1.   Open the image of the galaxy and select the *impose scale* option from the *analyse* menu. Impose the following parameters in the window which will open:
- Distance in pixel = Δpixel that you calculated previously,
- Real distance = $\Delta\lambda$ calculated previously,
- Unit of length = A (Angstrom)
2.   Below you can read the new scale of your image, which will be in pixel / Å.
N.B.: do not select *ok* to apply this scale to the image, but just note down the new scale, because the one you will need will be its reciprocal, i.e. Å/pixel:

$$\text{Scale} = \dots\dots\dots \text{ pixel / Å}$$

2.c Calculating shift Δλ

The deformation of the line in the spectrum caused by the Doppler effect, which produces the characteristic "S" shape, enables us to calculate the rotation velocity as a f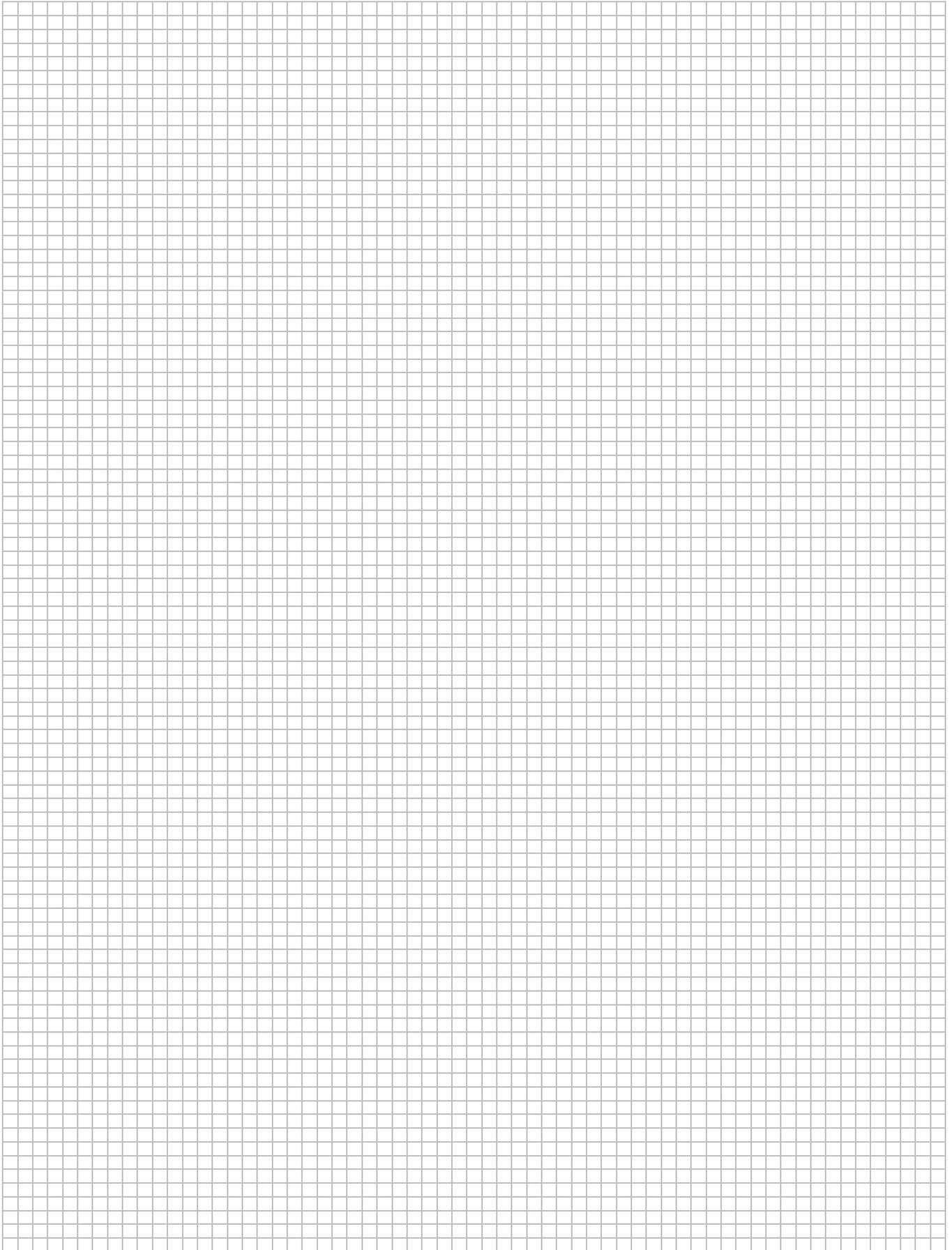unction of the distance from the centre of the galaxy. The velocity is proportional to the shift in wavelength; we therefore need to measure the Δλ at different distances from the centre.

1.   Enlarge the image to visualise it better by selecting *magnifying lens* from the toolbar and clicking on the image in the zone that you want to enlarge with the left-hand button of the mouse.
2.   To move the enlarged image and in order to get a better view of the lines of the gas, use the *scroll with mouse* key marked with the hand symbol. At this point, select the *select straight line* option and, after choosing from the *edit/options/graph options* menu, use the profile option and plot a horizontal line on the chosen emission line (to ensure that the line is straight, keep the shift key pressed down while you plot the selection). Mark in the table below the value of the distance from the centre of the line you have plotted, i.e. the value of y which appears in the bottom left-hand corner in the SalsaJ bar.
3.   Now select the *profile* option from the *analyse* menu. If you click on the centre of the peak of the profile, a table will appear showing the ordinate and abscissa values of the point on which you have clicked. Note down in the table below the position of the abscissa of the centre of the curve (the ordinates are grey values which do not concern us)
4.   Repeat this operation for different regions of the emission line, at various distances from the central horizontal line, and note down the values in the following table.

Centre Curve (in pixels)	Distance from centre (in pixels)

For the galaxy NGC 7083, whose images you will find in this site, the lines that we need to analyse are those of nitrogen doublets (NII) and sulphur doublets (SII), when λ_1=6564 Å, λ_2=6584 Å, λ_3=6713 Å e λ_4=6733 Å (these values will be important in the next phase).

Plot on the graph the position y of the selection on the line as an abscissa and the value of the centre of the profile as an ordinate.
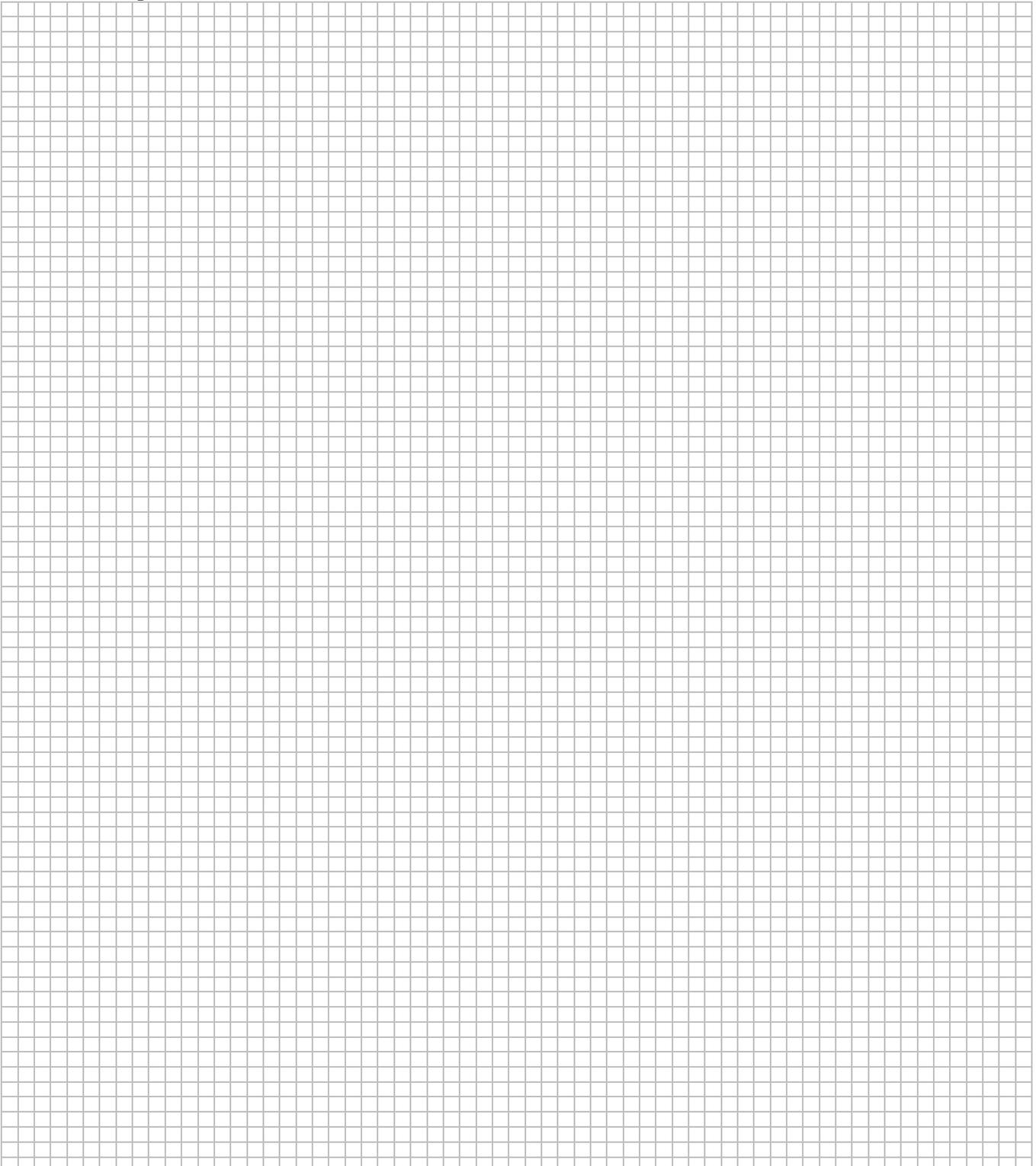
The graph thus obtained "resembles" the rotation curve of the galaxy, which is the graph of the rotation velocities as a function of the distance of the galactic centre *r*. **N.B.**: we would predict that the rotation of the galaxy is symmetrical to the centre of the graph since the rotation of the galaxy causes a part of it to come closer to us, while the other part moves away from us, but both at the same velocity.

Phase 3 : Calculating velocity

3.a Calculation of the Doppler Shift

It is now necessary to transform the quantity in abscissa into "distance from the centre of the galaxy" and the quantity in ordinate into "wavelengths" and then into "velocity".

To transform the coordinates of the centre of the line in pixels into wavelengths in Angstroms, we must consult the scale for the image that we found in phase 2.c. This scale is in pixel / Å; therefore performing the inverse operation should allow us to calculate how many Angstroms correspond to each pixel.

Lastly, to obtain the velocity, we must apply the formula of the Doppler effect: $\Delta\lambda / \lambda_{em} = v / c$, where $\Delta\lambda$ is the difference between our general coordinates and the coordinate of the centre of the line of emission, c is the speed of light (300000 km/s).

As regards the axis of the abscissa, the distance in pixels needs to be converted into arcseconds: this depends on the scale of the sensor with which the images have been obtained (in the case of the ESO 1.52m instrument, the scale is 0.82 arcsecs/pixel); this value shows which angle is being "observed" by the sensor; in other words, how many arcseconds correspond to each pixel. The graph shows the difference with respect to the centre of the line of emission.

Distance from centre (arcsec)	$\Delta\lambda$	Velocity (Km/s)

3.b Relation $v_{oss} = v_{rot}\ sen(i)$

Since the galaxies possess a slight inclination with respect to the line of sight, the radial velocity is linked to the actual rotation velocity of the relation $v_{oss} = v_{rot}$ sen i, where i is the angle that the plane of the galaxy forms with the celestial sphere. For this reason, the deprojected rotation velocity will be: $v_{oss} / sen(i) = v_{rot}$.

Note that the curve you obtain is symmetrical to the centre of the galaxy (we would expect that the velocity of distancing and approaching of two points situated at the same distance from the centre of the galaxy, though at opposite points to it, would be the same!).*(cf. fig 3.1)*
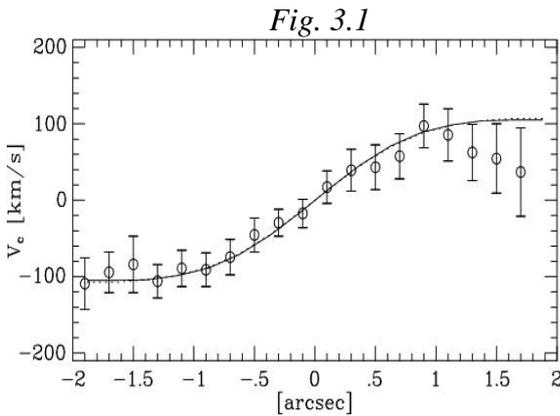
Fig. 3.1	*Fig. 3.2*

Up-folded curve

Therefore if we consider the absolute value of the velocity we obtain a graph such as the one in figure *3.2* (with two points for extrapolating the profile of the curve)

This is your rotation curve, which tells you that the nearest parts of the axis of rotation, i.e. closest to the centre of the galaxy, rotate slowly, whereas as we gradually move away from the centre, the velocity increases until it reaches its maximum value in the outermost parts of the galaxy.
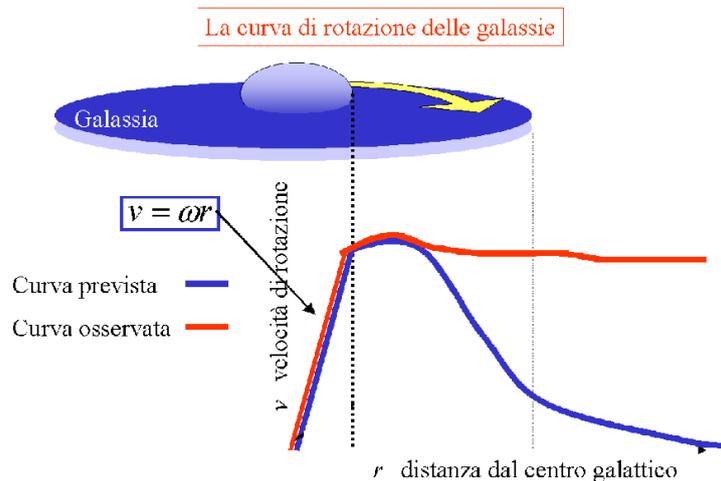
The predicted curve and the observed curve are rather different!

Let's analyse the profile that we would expect. As long as we measure the rotation velocity of the gas at small distances (i.e. still within the nucleus of the galaxy), assuming that the nucleus rotates like a rigid body at a constant angular velocity, we would expect the velocity to increase with the increase in distance from the centre of the galaxy:

$$v = \omega \cdot r$$

When we "emerge" from the nucleus, it is legitimate to believe, as a first approximation, that the whole mass is concentrated almost exclusively in the nucleus of the galaxy, where there is a high concentration of stars and therefore that the law of universal gravity can be applied: the general element of mass m at distance r is subject to the force of Newtonian gravity, in other words it undergoes the attraction of the mass M contained in radius r; thus the second principle of dynamics $F=ma$ is written:

Newton force = m • centripetal acceleration

or

$$G \cdot m \cdot M / r^2 = m \cdot v^2 / r$$

from which it can be seen that with the increase of the distance r we would expect that velocity would begin to decrease as the square root of r. Therefore the expected curve is the type shown in blue in the figure.

At large distances from the centre, though, what we observe departs from this expected pattern since the measured speeds at a large r are higher than predicted.

In general, if we consider solid bodies orbiting around a mass, the rotation velocity of the bodies increases as the central mass increases. A rapid rotation velocity requires large central masses. The high velocities that can be observed in the rotation velocities of the galaxies can be explained if it is assumed that the mass within the radius r is much greater than the "visible" one (in the sense of it being calculated from the observations!): in other words, we should suspect that there is matter that cannot be "seen", but whose effects can be seen on the rotation of the outer parts of the galaxy. This is the famous concept of *dark matter*, whose precise nature is still the subject of debate among astronomers; it can be calculated from an analysis of the rotation curves (and using other methods that confirm this hypothesis) that the matter which we cannot "see" represents 96% of the Universe!

Phase 4 : Calculating the total mass of the galaxy

In order to calculate the mass of a spiral galaxy, it is assumed that the galaxy is a stellar system in dynamic equilibrium and the main force agent acting upon it is that of gravitational attraction.

On the basis of these assumptions, in the fame of reference of the galaxy, one can presume that there is equilibrium between the force of gravitational attraction and centrifugal force: GmM/r^2 $=mv^2/r$, where M is the mass of the galaxy within the radius r, v is the velocity of the generic "piece" of galaxy of mass m and G is the constant of universal gravity.

The only unknown in this set of relations is the mass of the galaxy M, which can be expressed as:

$$M = \frac{v^2 r}{G}$$

where r is the last value in abscissa to which the rotation curve extends and the velocity is the greatest of the curve.

The mass thus calculated is not sufficient to justify the high velocities which are measured in the outer regions of the galaxy.

Useful information

Velocity of the galaxy = 3101 km/s

Inclination of the galaxy = 53 degrees

Calibration lamp: