Seven steps for a dwarf star

From Doppler to exoplanets

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Exercice proposed by :

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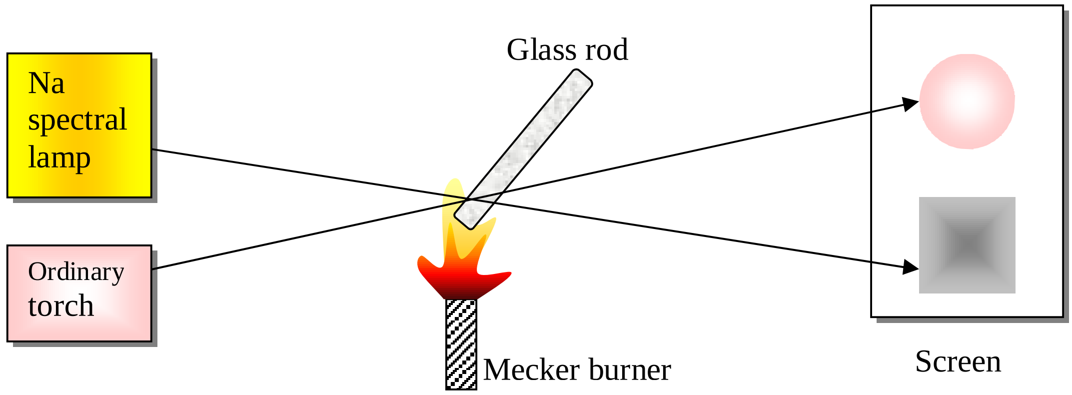
1^{st} STEP : SPECTROSCOPY

1.1. A star emits a continuum spectrum due to nuclear reactions, with absorption lines due to the atmosphere of the star.

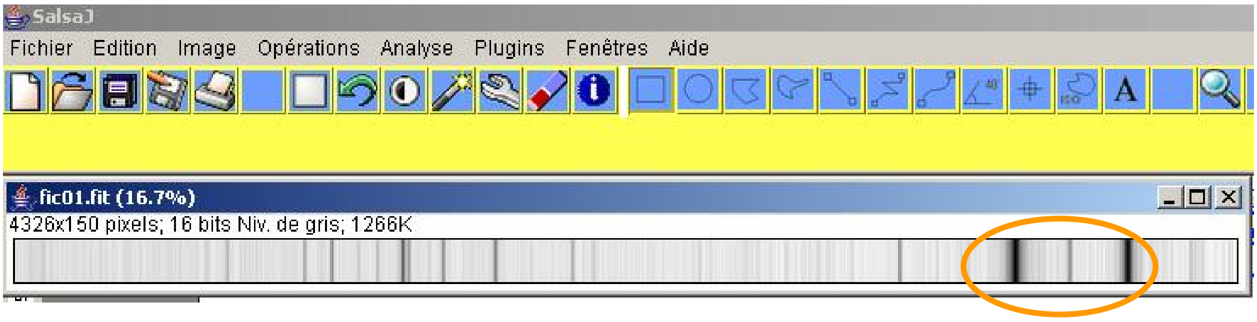
We shall visualize 11 spectra of a binary star, at different dates ; we study the part of spectrum around Na lines : we choose Na lines because you can have easy experiments with Na lamps or with ordinary kitchen salt.

1.2.Experiment :



The glass rod contains NaCl ; when highly heated, it produces Na light that absorbs the light from Na spectral lamp, whereas torch light looks unchanged. This phenomenon is called **Na lines resonance.**

1.3 Open image fic01.fit



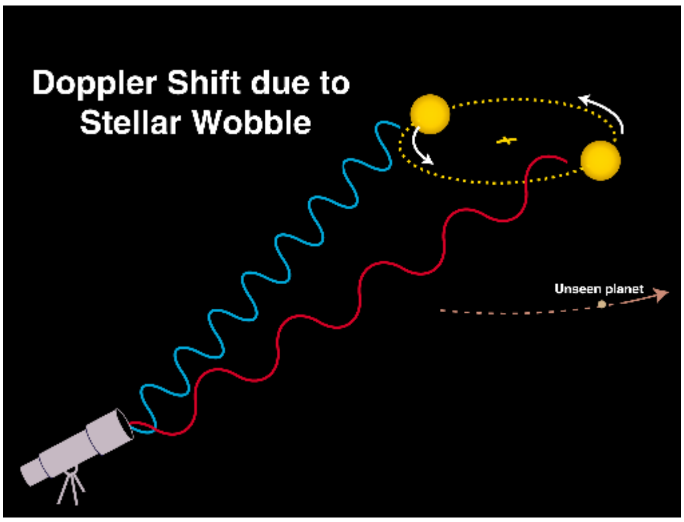
Na double line

We have 11 images.fit, at the 11 following dates

Spectrum number	Date (days)
1	0
2	0.974505
3	1.969681
4	2.944838
5	3.970746
6	4.886585
7	5.924292
8	6.963536
9	7.978645
10	8.973648
11	9.997550

The interval between two dates is roughly one day.

Each object in a binary system moves around the barycenter. So, the spectrum lines move according with time (Doppler shift).



2 - ANIMATED MOTION OF SPECTRUM LINES

USE images.fit

Note : images.fit (fit =fits = fts) are available for animation
images.dat are available for optical spectra

Here is the procedure to have a global view of Doppler shift when the star moves around barycenter :

Open Salsa J(click on SalsaJ icon)

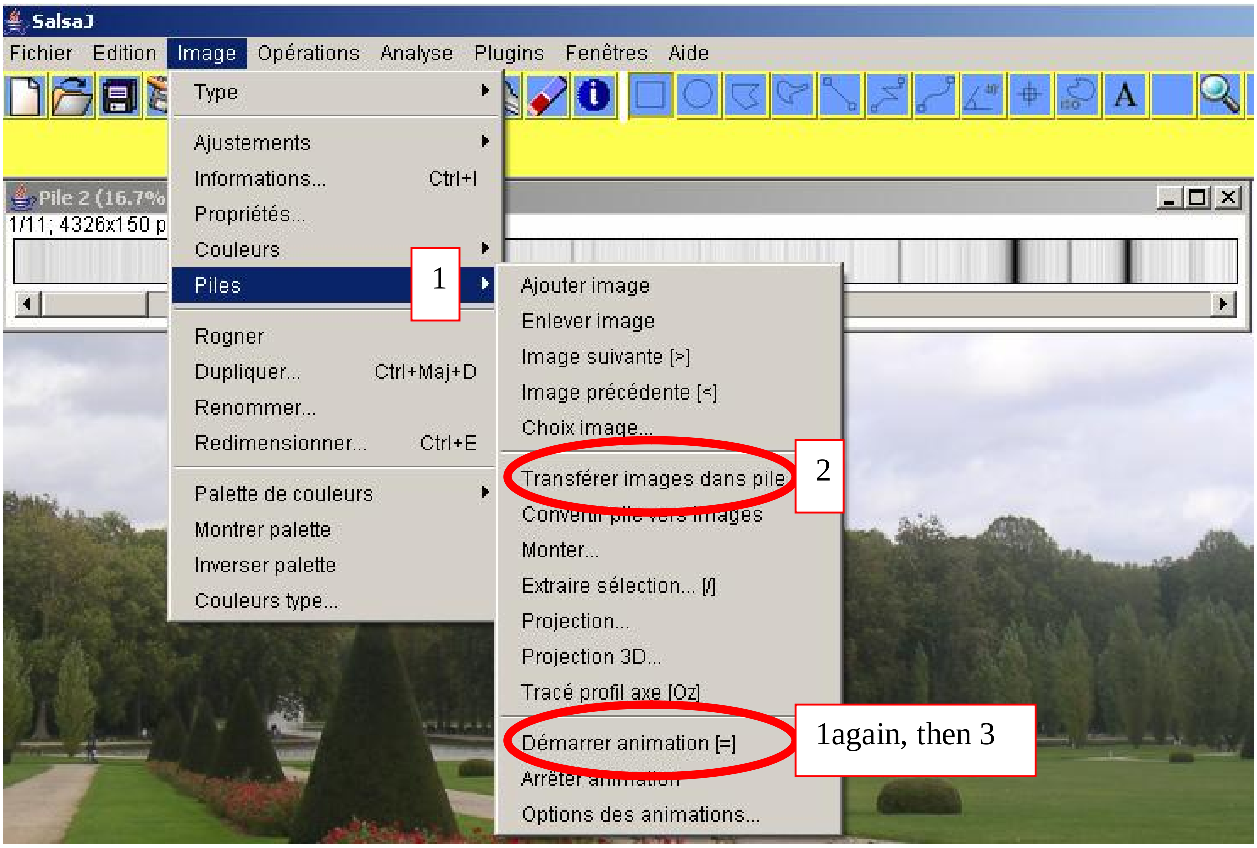
Click on **File (Fichier),** then **Open** (**Ouvrir)** in the pull-down menu

Enter folder (dossier) **:** ***binary system***

Select the 11 spectra images.fit from fic01.fit to fic11.fit : press Shift to select the 11 images.fit at once)

Open (Ouvrir) these 11 images, then
click on **Images** : you get a roll-down menu ;
click on **Stacks (=Piles)** : you get a new menu ;
click on **Transfer Images to Stacks** (=**Transférer images dans Pile)**

Click again on **Images / Stacks (=Piles) / Démarrer animation**



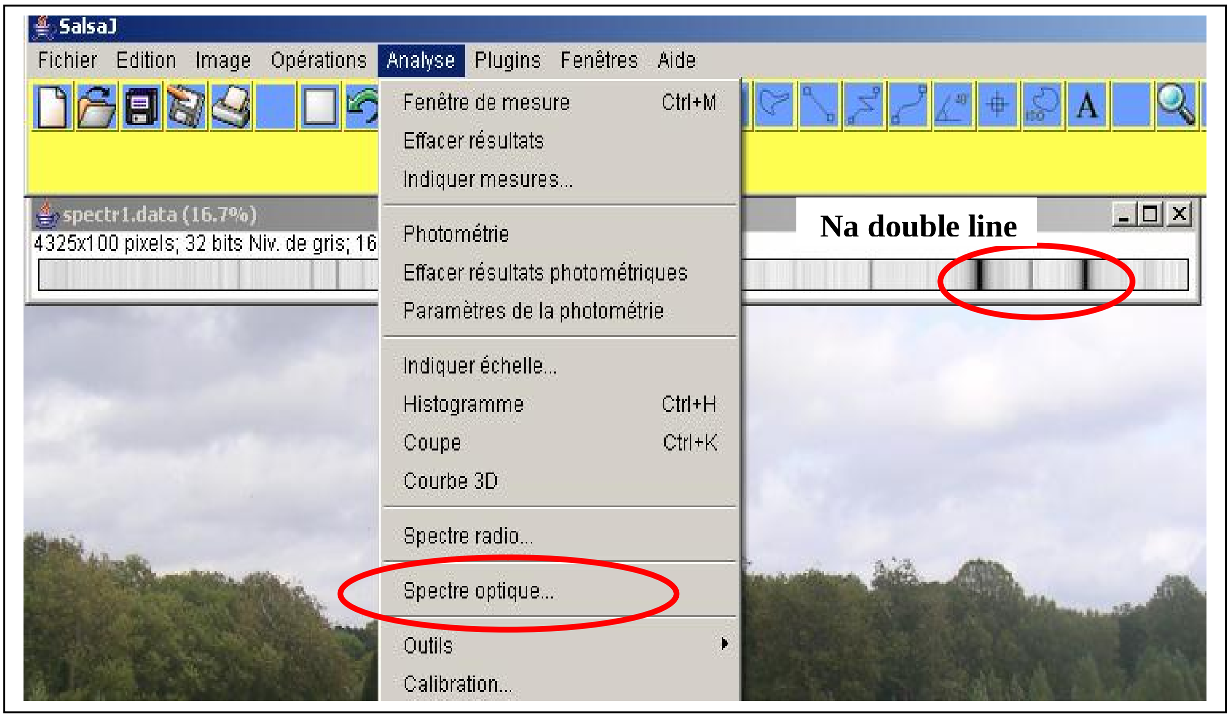
Enjoy the Doppler shift during the rotation of the star around the barycenter of the binary system, then close.

3. MEASURE WAVELENGTH λ AND FLUX, OPTICAL SPECTRUM

Use images.dat

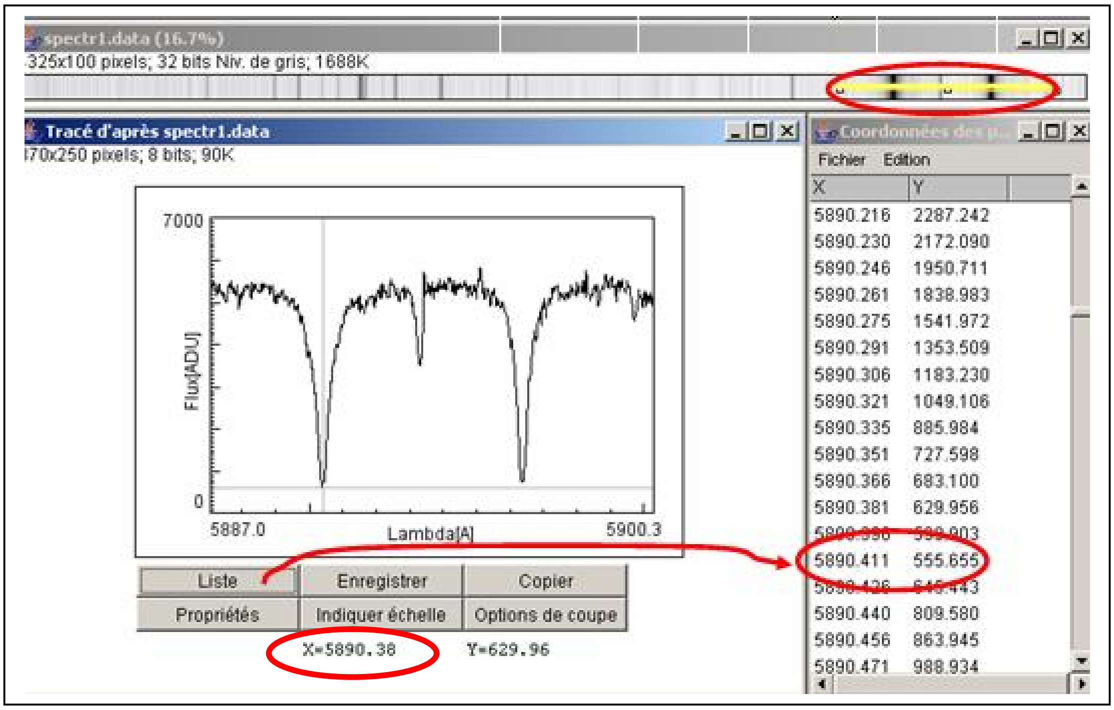
3.1. Investigation of spectrum 1 : spectr 1.data

Click on **Analysis / Optical Spectrum**/ binary system / spectr1.data



3.2. Flux according to wavelength : Φ= f(λ)

Spect1.dat image / Click on the **Straight line selection (Sélection rectiligne)** icon, then draw a straight line **across the Na doublet** (to have an horizontal line, press Shift during the drawing) / Click on **Analysis / Plot Profile** (=Coupe) : you get Φ= f(λ)



You notice :

Two deep absorption lines of Na
A smaller one (Ni I) ; we shall not use this one

With mouse (or equivalent) measure the absorption Na doublet wavelengths (Å = 10^{-10}m)

See X under the curve or inside the list :

$$\lambda_1 = 5890{,}411 \text{ Å} \qquad \lambda_2 = 5896{,}366 \text{ Å}$$

Compare these values to reference values (Na lines in our laboratory) :

$$\lambda_{Na\ D1} = 5889{,}950 \text{ Å} \qquad \lambda_{Na\ D2} = 5895{,}924 \text{ Å}$$

The difference between reference and measured value is the Doppler shift.

3.3. Eleven spectra : spectr i.data, i from 1 to 11

Follow same procedure as with spectr 1.data

List of results :

Spectrum	Date t (days)	λ_1 (Å)	λ_2 (Å)
1	0	5890,411	5896,366
2	0.974505	5890,496	5896,511
3	1.969681	5890,491	5896,446
4	2.944838	5890,305	5896,274
5	3.970746	5890,014	5896,029
6	4.886585	5889,815	5895,800
7	5.924292	4889,642	5895,597
8	6.963536	5889,638	5895,621
9	7.978645	5889,764	5895,793
10	8.973648	5890,056	5896,042
11	9.997550	5890,318	5896,303

Note : Interpolation may help to improve accuracy.

4TH STEP: CALCULATING RADIAL VELOCITY OF THE STAR WITH DOPPLER SHIFT

$$\Delta\lambda / \lambda = vrad/c$$

$\Delta\lambda_i = \lambda i - \lambda_{Nai}$; i = 1 ou 2, λ_{Nai} reference wavelenth et λi measured wavelength of Na in the moving star spectrum.

vrad = velocity projection of the staron the line of sight; it includes barycentre speed+ motion of the star around the system barycentre.

c = speed of light

Using Na D1 (λ_1) :

Spectrum	Date t (days)	$\lambda_1 - \lambda_{Na1}$ (Å)	$V_E = c \cdot (\lambda_1 - \lambda_{Na1}) / \lambda_{Na1}$ (km/s)
1	0	0.461	23.48
2	0.974505	0.546	27.81
3	1.969681	0.541	27.56
4	2.944838	0.355	18.08
5	3.970746	0.064	3.26
6	4.886585	-0.135	-6.88
7	5.924292	-0.308	-15.69
8	6.963536	-0.312	-15.89
9	7.978645	-0.186	-9.47
10	8.973648	0.106	5.40
11	9.997550	0.368	18.74

You can do the same with Na D2 ; if you work very accurately, you will notice some differences between the two lines results ; so you will improve the accuracy by using both lines and averaging.

Accuracy : with one line : 4,2 %

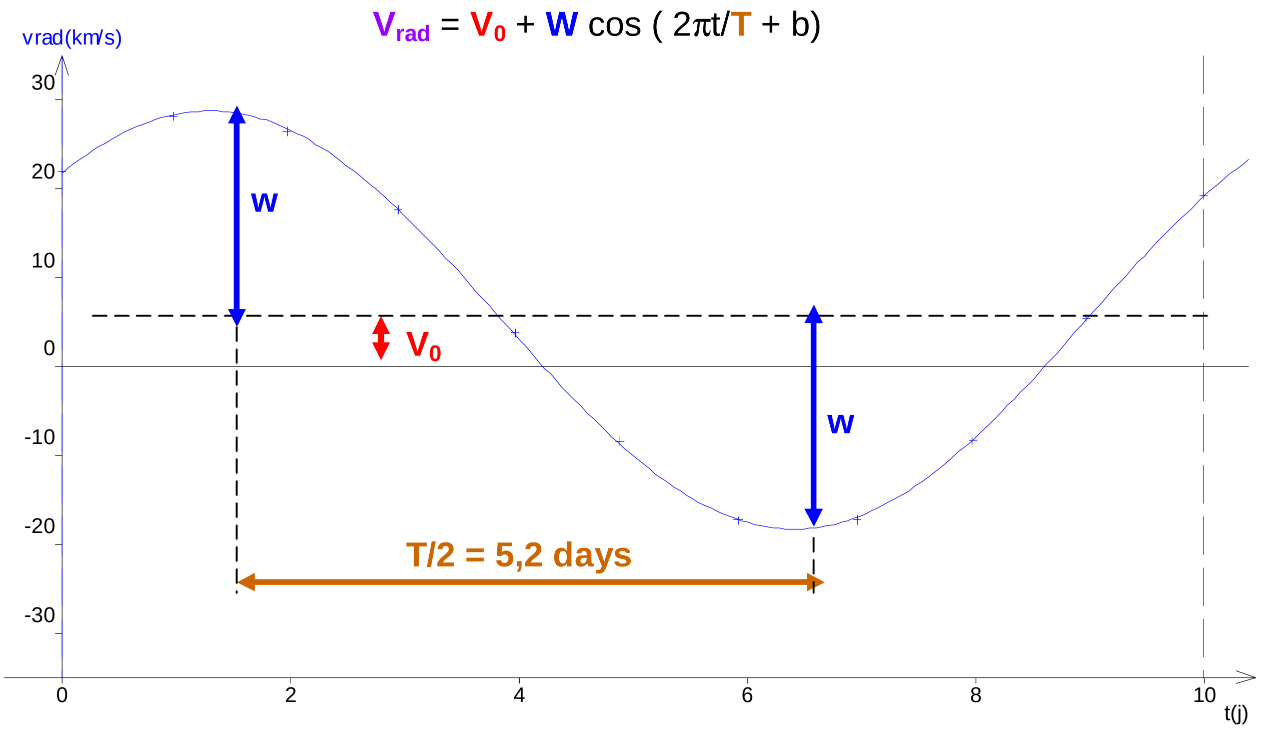
With two lines : 2%

The more lines you will use, the better accuracy you will get. This will be useful for measuring small velocities, such as radial velocity due to exoplanets.

5 TH STEP : RADIAL VELOCITY OF THE STAR , AS A FUNCTION OF DATE

With a spreadsheet, Regressi (there is a free version on the web), Excel …, we propose a model for $v = V_E = f(t)$.
With Regressi for example, you enter variables t (day) et $v = V_E$ (km/s)
We try : $v_{observée} = V_{rad} = V_0 + W * \cos((2*\pi*t/ T)+ b))$
OK

Then click on Adjust (**Ajuster)**, noting T =10.34 jours



With our model, we get :

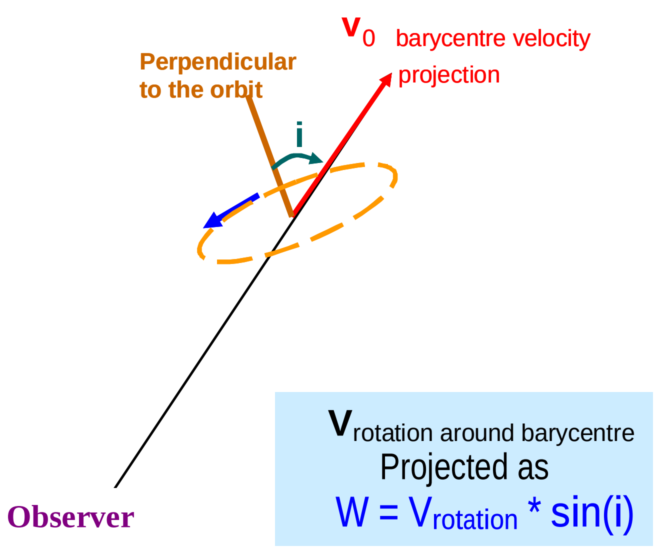
V_0 = 5.9 km/s **W = 23.2 km/s**

Precision : 4.4%

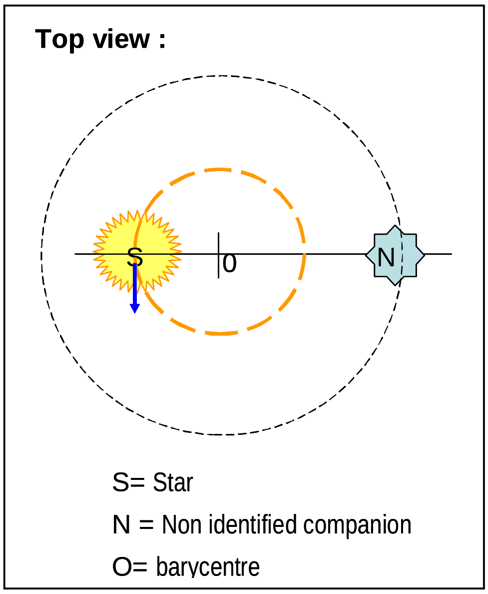
Because of angle i between perpendicular to the orbit and line of sight, W is a lower limit of the rotating velocity of the star :

$$V = W / \sin i$$

We shall proceed with sin(i) = 1.



6TH STEP : DETERMINING THE MASS OF THE COMPANION IN BINARY SYSTEM



Mass of the Earth, telluric planet: $M_T = 6\ .10^{24}$ kg
Mass of Jupiter, giant planet: $M_J = 2\ .10^{27}$ kg
Mass of the Sun : $M_S = 2{,}0.10^{30}$ kg

<u>We suppose circular orbits and we use Kepler law</u>

The visible star S mass is M
The companion star mass is m
We call O the barycentre of the binary system.
We want to calculate the mass m of the companion star.

- **Kepler's law :**
  $\mathbf{T^2/(SN)^2 = 4\ p^2/[G\ (M_S + m_N)]}$
- **Barycentre:**
  $\mathbf{SN = [(M_S + m_N)\ /m_N]OS}$
- **Circular orbit :**
  $\mathbf{v_{Star} = 2\ p\ OS/T = W\ /\ \sin(i)}$ **; we take sin(i) = 1**

Hence:

$$\mathbf{2p\ G\ m_N^{\ 3} = v_{star}^{\ 3}\ T\ (m_N + M_S)^2}$$

You can also write : $r\ [(W.\ T\ /\ 2\pi \sin i\ ) + r\ ]^2 = G\ M\ T^2\ /\ 4\ \pi^2$

DATA : $G = 6.67.\ 10^{-11}$ uSI ; sin i = 1
$W = 23.1$ km/s ; $T = 10.34$ jours $\approx 9.0.10^5$ s
$M = 1.05\ M_{solaire} = 1{,}05\ .2{,}0.10^{30} = 2{,}1\ .10^{30}$ kg

Computation with pocket calculator [sin(i) = 1, and circular orbit], we get

$\mathbf{m_N = 0.275\ M_{Star} = 5{,}8\ .\ 10^{29}}$ **kg**

$R = OE = 3{,}31\ .10^9$ m $\quad r = ON = 0.275\ R = 1{,}2\ .10^{10}$ m

The Companion is a dwarf star that emits almost no visible light ! **J**

NB : Sin i = 1 undervalues R. If sin i is lower, R increases, r decreases, hence m increases.
So, sin i = 1 gives a lower limit on m.

7TH STEP : DISCOVERING AN EXOPLANET WITH DOPPLER SHIFT OF A STAR

Data :

Mass of the Earth, telluric planet: $M_T = 6\,.10^{24}$ kg
Mass of Jupiter, giant planet: $M_J = 2\,.10^{27}$ kg
Mass of the Sun : $M_{Sun} = 2{,}0.10^{30}$ kg

Mass of a telluric planet = $M_{Sun}/10^6$
Mass of a giant planet = M_{Sun} / 1000

Radial velocity variation
due to a telluric planet : mm/s to dm/s
due to a giant planet : m/s to 100 m/s

Rotation periods of exoplanets : 3 to 3000 days

Rough estimate :

For an exoplanet, even a giant one, mass m is far lower than a star mass M ; we can then approximate :

$$2p\ G\ m_{planet}^{3} = v_{star}^{3}\ T\ (m_{planet} + M_S)^2$$

$m = K.\ V\ .\ T^{1/3}.\ M^{2/3}$ with K constant = $(1/\ 2\pi G)^{1/3}$

The higher exponant above is 1 (V^1) , and only 1/3 for T, 2/3 for M ; so, the main factor is V

A 1000 times smaller speed induces a 1000 times lower mass m, whereas a 1000 times lower period induces a 10 times lower mass m .

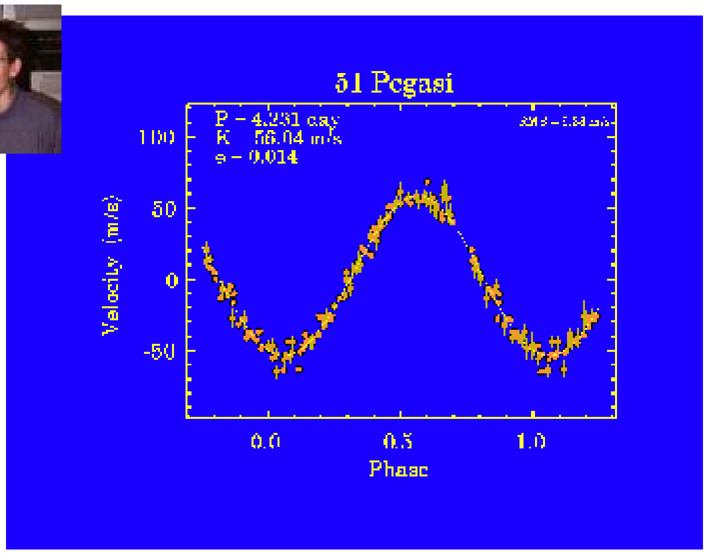
Discovery of the first exoplanet
(Mayor, Queloz and team, 1995)

51-Pegase star mass M ≈ $M_{sun} = 2{,}0.10^{30}$ kg
Radial velocity of 51 Pegase : **V = 60 m/s**
Period of 51 Pegase : T = 4,2 days.

Companion called 51 Pegasi B orbits the star .

Using : $m_{companion} = K.\ V\ .\ T^{1/3}.\ M_{Star}^{2/3}$
with K = $(1/\ 2\pi G)^{1/3}$

We get : $m_{Companion} = 9{,}1.10^{26}$ kg soit 0.45 $M_{Jupiter}$



This contributes to prove that 51-Pegasi B is a giant exoplanet.

The Doppler method to detect exoplanets needs thousands of lines in the spectrum of the star to reach a m/s accuracy : it works for giant exoplanets.
We cannot yet (2006) detect a Doppler shift due to a telluric exoplanet : radial velocity is too small !!